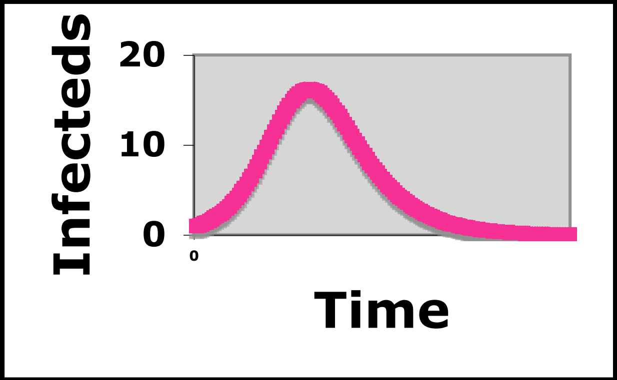
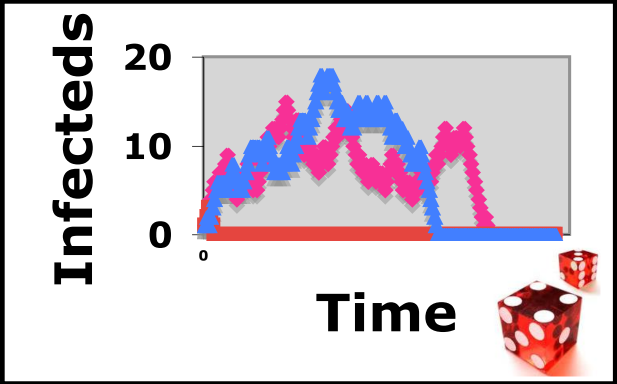
Probability and
the importance of randomness

Deterministic versus stochastic

- All the models so far have been deterministic
- Deterministic models give same output for the same starting conditions
  - no element of randomness or chance
- Stochastic models include an element of chance





Interpreting rates

- Let's consider the recovery rate σ

Interpreting σ=0.1…

…if many infecteds, we are happy that 0.1 or 10% of the infected population will recover per time unit

…but, if just a few infecteds, the deterministic interpretation implies that 0.1 of an individual could recover (a little odd!)

Interpreting rates

- Let's consider the recovery rate σ

Deterministic models do not handle dynamics at the individual level well!

Stochastic or probabilistic models will help us capture dynamics at the individual level

Thinking probabilistically

- From the perspective of the individual a better interpretation of $\sigma=0.1$ is

Each individual has a 10% chance
(a probability of 0.1)
of recovery in time interval of 1 **

** The relationship between rates and probabilities is actually slightly more complicated but this is a very good approximation for low rates.

How do we capture this?

- We can incorporate probabilities into simulations using a random number generator

Random numbers in R

Requesting 1 random number between 0 and 1

Requesting 3 random numbers between 0 and 1

```
>
> runif(1)
[1] 0.07053163
> runif(1)
[1] 0.2868392
> runif(3)
[1] 0.4336072 0.8995862 0.1137983
> runif(3)
[1] 0.5798326 0.5079621 0.6202416
>
>
```

Using random numbers

Example: prob of recovery =0.1

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1. Pick a random number between 0 and 1

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Example: prob of recovery =0.1

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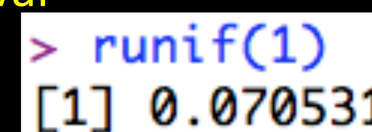
2. If less than 0.1, the individual recovers in this time interval



Using random numbers

Example: prob of recovery =0.1

1. Pick a random number between 0 and 1


```
>
> runif(1)
```

2. If less than 0.1, the individual recovers in this time interval


```
> runif(1)
[1] 0.070531
```

3. If greater than 0.1, the individual does not recover


```
> runif(1)
[1] 0.286839
```

Using random numbers in practice

- At each time step, could repeat procedure for each of the I infecteds
- Not the most efficient method
- Use the binomial distribution in R to do all of the I infecteds at once

Refresher on the binomial distribution

Pie example

- The probability that a pie is contaminated with a bacterium is 1 in 10
  - Prob(pie contaminated) = 0.1

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Prob(both contaminated)= $(0.1)^2$ =0.1*0.1 =0.01

Pie example

- The probability that a pie is contaminated with a bacterium is 1 in 10
  - Prob(pie contaminated) = 0.1

- What is probability that two pies selected at random are not contaminated?

Prob(both <u>un</u>contaminated)= $(1-0.1)^2$ =0.9*0.9 =0.81

Combining probabilities

Prob(neither contaminated) = $(1-0.1)^2$ = 0.81

Prob(both contaminated) = $(0.1)^2$ = 0.01

Sum of probabilities 0.82

Combining probabilities

Prob(neither contaminated) = $(1-0.1)^2$ = 0.81

Prob(one contaminated)= (1-0.1)*0.1+0.1*(1-0.1) = 0.18

Prob(both contaminated) = $(0.1)^2$ = 0.01

Sum of probabilities 1.0

Combining probabilities

Prob(neither contaminated) = $(1-0.1)^2$ = 0.81

Prob(one contaminated)= $2*(1-0.1)*0.1$ = 0.18

Prob(both contaminated) = $(0.1)^2$ = 0.01

Sum of probabilities 1.0

Combining probabilities

2 pies means 3 outcomes

Prob(neither contaminated) = $(1-0.1)^2$ = 0.81

Prob(one contaminated)= 2*(1-0.1)*0.1 = 0.18

Prob(both contaminated) = $(0.1)^2$ = 0.01

Sum of probabilities 1.0

Probability distributions

- Suppose we looked at 10 pies each with probability p=0.1 of being contaminated

- Now, there are 11 possible outcomes

  - P(0 contaminated)= $(1-0.1)^{10}$
  - P(1 contaminated)= $10*0.1*(1-0.1)^{9}$
  - P(2 contaminated)= ……
  - ….
  - P(10 contaminated)= $(0.1)^{10}$

- These probabilities form a binomial distribution

Probability distributions

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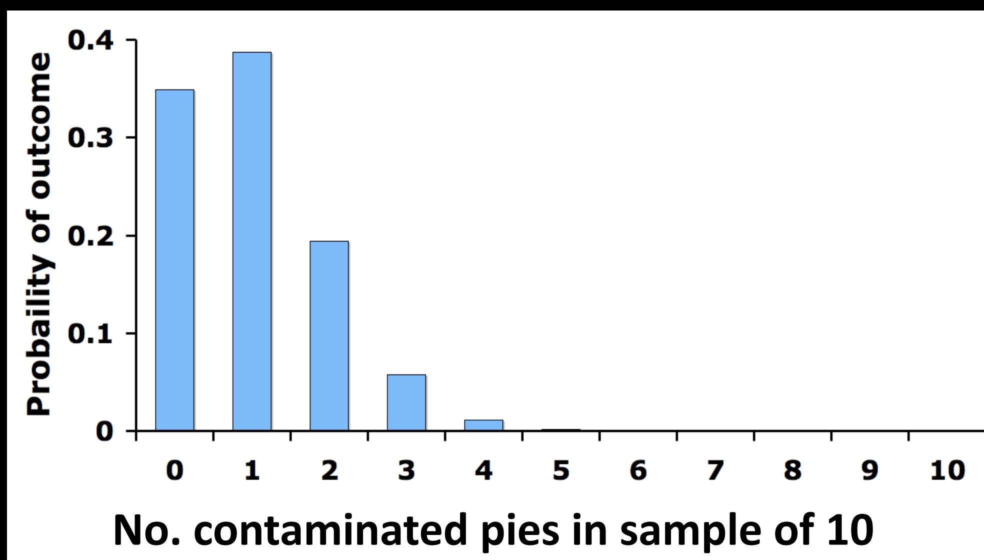


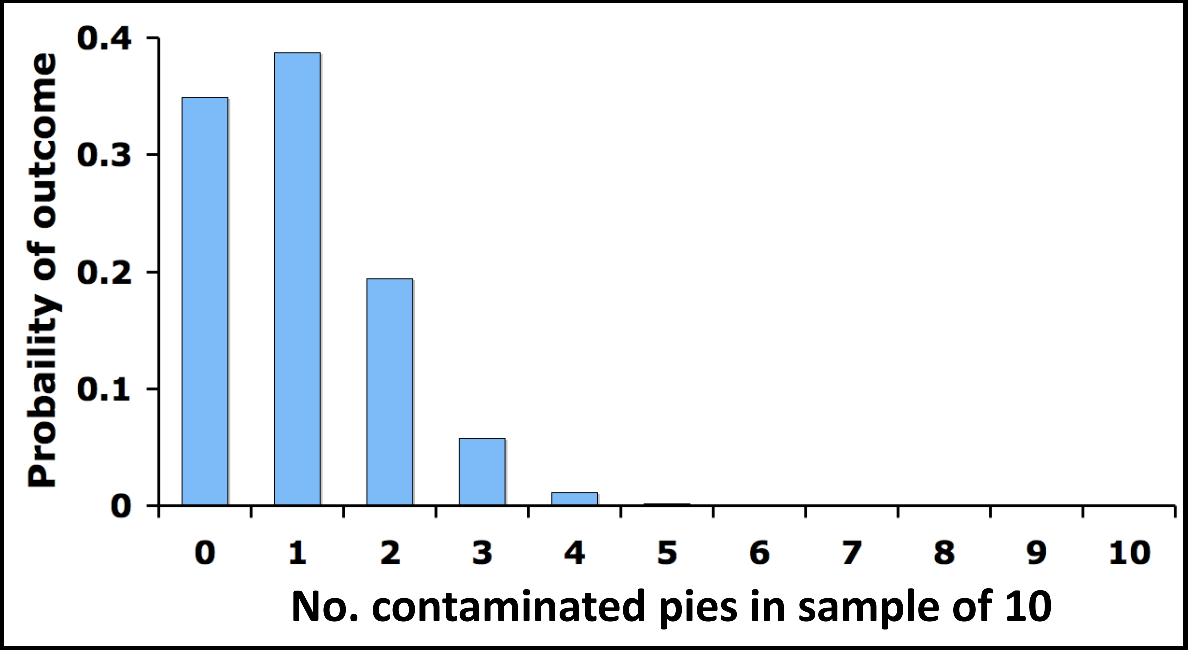
- These probabilities form a binomial distribution

The binomial distribution

- Shape of distribution determined by the number of trials, n, and the probability of success, p
  - a classic example is tossing a coin n times, with probability of a head p=0.5
- For our meat pie example n=10 and p=0.1

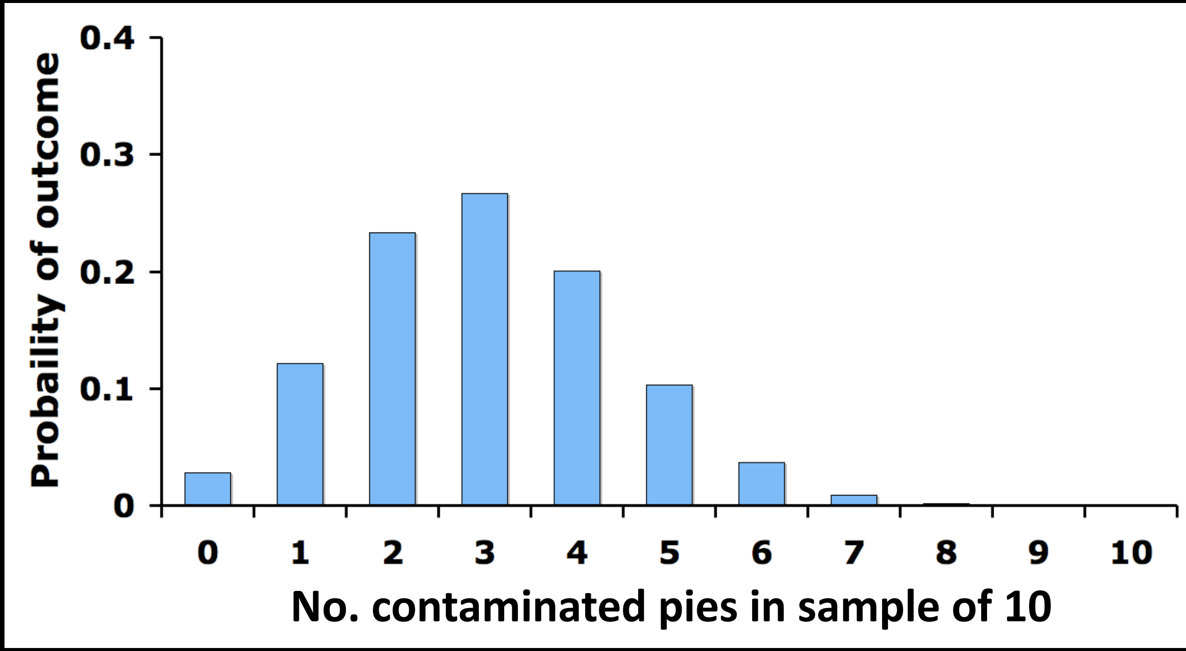
Number of contaminated pies

- Binomial distribution for n=10, p=0.1



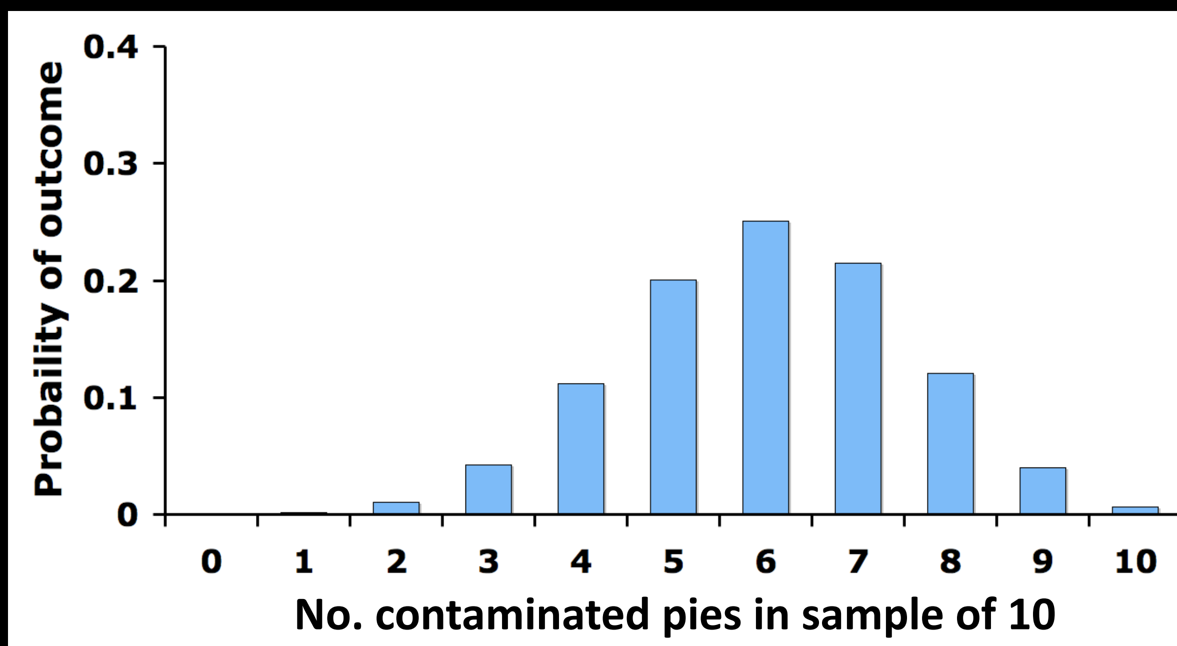
Number of contaminated pies

- Binomial distribution for n=10, p=0.3



Number of contaminated pies

- Binomial distribution for n=10, p=0.6



Binomial distribution summary

- Defined by Binomial(n,p)
  - n = number of trials
  - p = probability of success in each trial
- Can sample from distribution in R
  - rbinom(1, 10 , 0.1)

```
>
> rbinom(1,10,0.1)
[1] 1
> rbinom(1,10,0.1)
[1] 0
> rbinom(1,10,0.1)
[1] 2
> rbinom(1,10,0.1)
[1] 2
> rbinom(1,10,0.1)
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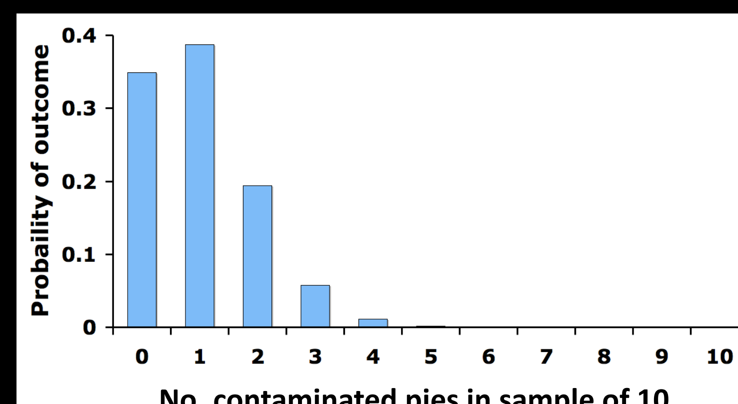
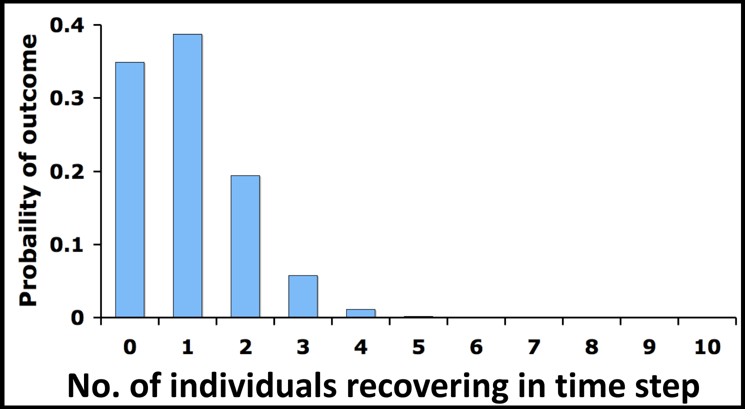


End of refresher

Using binomial to simulate recovery

- Binomial dist defined by
  - n= num trials
  - p= probability



- To simulate recovery
  - n = I (number infecteds)
  - p = σ (recovery probability)

```
>
> sigma <- 0.2
> I <- 12
>
> Number.recovering <- rbinom(1, I, sigma)
> Number.recovering
[1] 1
>
> Number.recovering <- rbinom(1, I, sigma)
> Number.recovering
[1] 3
>
```

Same trick for transmission

- Look at the rates
  - total rate of transmission is $\beta SI/N$
  - the rate per susceptible is $\beta I/N$

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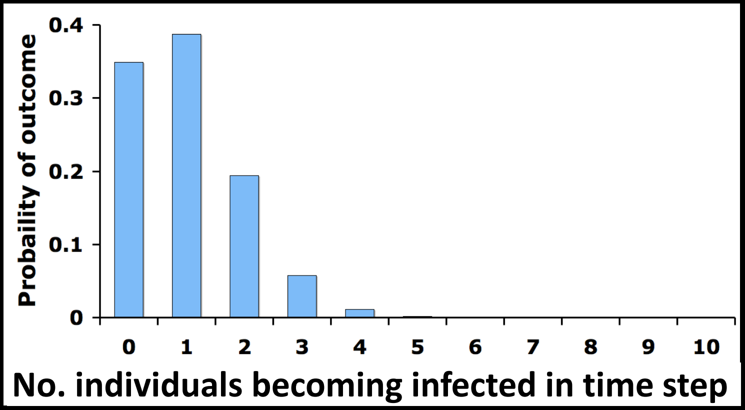
- Look at the rates
  - total rate of transmission is $\beta SI/N$
  - the rate per susceptible is $\beta I/N$

- Interpret rate per individual as a probability $p=\beta I/N$ of infection

Using binomial to simulate transmission

- Defined by Binomial(n,p)
  - n=S (susceptibles)
  - p=βI/N (prob of infection)

- Can sample from distribution in R
  - rbinom(1, S , βI/N)



Using binomial to simulate transmission

- Defined by Binomial(n,p)
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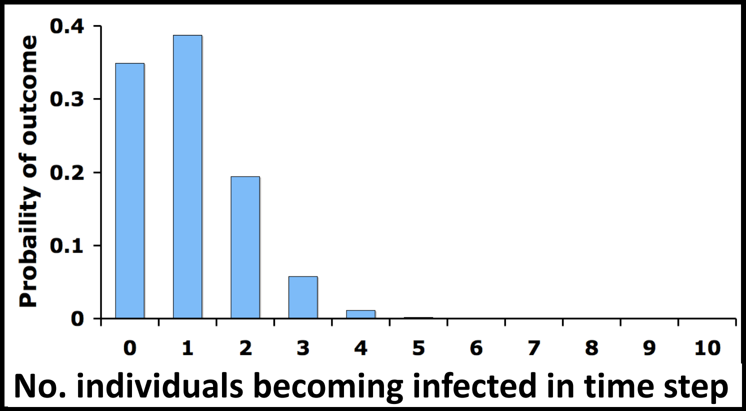


```
>
> N <- 100
> I <- 50
> S <- 4
> beta <- 0.2
>
> Number.getting.infected <- rbinom(1, S, beta*I/N)
> Number.getting.infected
[1] 0
> Number.getting.infected <- rbinom(1, S, beta*I/N)
> Number.getting.infected
[1] 0
> Number.getting.infected <- rbinom(1, S, beta*I/N)
> Number.getting.infected
[1] 1
>
```

The impact of randomness

- Variety of outputs for same starting conditions
  - in contrast to a deterministic model
- Extinction in outbreaks even if $R_0>1$!
- Observed stochasticity decreases when
  - population size is large
  - initial number of infecteds large